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Anomaly Mediation, Fayet-Iliopoulos D-terms and the Renormalisation Group

R. Hodgson, I. Jack, D.R.T. Jones

Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

Abstract

We address renormalisation group evolution issues that arise in the Anomaly Mediated Supersymmetry Breaking scenario when the tachyonic slepton problem is resolved by Fayet-Iliopoulos term contributions. We present typical sparticle spectra both for the original formulation of this idea and an alternative using Fayet-Iliopoulos terms for a U_1 compatible with a straightforward GUT embedding.

1 Introduction

Anomaly mediation (AM) [1]- [24] as the main source of supersymmetry breaking is an attractive idea. In AM, the soft supersymmetry-breaking $\phi^*\phi$ masses, ϕ^3 couplings and gaugino masses are all determined by the appropriate power of the gravitino mass multiplied by perturbatively calculable functions of the dimensionless couplings of the underlying supersymmetric theory. Moreover these functions are RG invariant; that is, their renormalisation scale dependence is correctly given by the renormalisation scale dependence of the dimensionless couplings. To put it another way, the AM predictions are UV-insensitive [18].

In recent papers we have explored a specific version of AM, where the tachyonic slepton problem characteristic of a minimal implementation of AM is solved by means of an additional U_1 gauge symmetry, U_1', that is broken at high energies. The scale of this breaking may be set by a Fayet-Iliopoulos (FI) D-term [23] or via dimensional transmutation [24]. In the former case we showed how it is quite natural for the effects of U_1' to decouple at low energies apart from contributions to the scalar masses, of the form of U_1' FI terms, which are automatically of the same order as the AM ones. In the latter we argued that it was possible to dispense with an explicit FI term, generating the U_1' breaking scale via dimensional transmutation, exploiting a flat D-term direction. In our explicit model, the low energy theory consisted of the usual MSSM fields with an additional gauge singlet chiral supermultiplet which is weakly coupled to the MSSM fields[1]; the possible cosmological and phenomenological implications (in particular the possibility that its fermionic component might be the LSP) remain to be discussed. In this paper we will confine ourselves to the first possibility, where the low energy theory simply consists of the MSSM fields.

In both the above scenarios, there is, however, a subtlety with regard to the afore-mentioned RG invariance, concerning the Fayet-Iliopoulos term associated with the SM (or MSSM) U_1, U_1^{SM}. Suppose for simplicity the U_1' breaking scale coincides with the gauge unification scale M_X, and that the U_1^{SM} FI term is zero there. It turns out that the presence of the U_1' FI terms in the effective field theory means that even though it is zero at M_X, the U_1^{SM} FI term can become significant in the evolution to low energies. Thus there will be contributions of FI form for both the U_1' and the U_1^{SM} to the scalar masses. Now as emphasised in Ref. [20], these two contributions can be reparametrised into a contribution of the form of a single U_1'' FI-term. It should now be clear, however, that the resulting form of this contribution will be a function of scale since the size of the U_1^{SM} FI term generated is a function of scale.

The upshot is that if we choose a U_1' with charges for the lepton doublets and singlets chosen so as to solve the tachyonic slepton problem, and also zero FI term for U_1^{SM} at M_X, the resulting spectrum will correspond to a nonzero FI term for U_1^{SM} at M_Z, *or* a zero FI term for U_1^{SM} at M_Z with a different pair of U_1' leptonic charges.

In this paper we shall firstly explain this issue in some detail and then repeat some of the precision calculations of Ref. [23] but now imposing boundary conditions at M_X, and taking the opportunity to update input values and correct some minor bugs in our previous analysis.

In the second part of the paper we consider a variation of the same idea where we augment

[1] The existence of this light field is in fact a consequence of general arguments concerning AM decoupling given by Pomarol and Rattazzi [3].

the theory in a minimal way so as to render the U_1' charge assignments compatible with a GUT embedding; specifically SU_5, SO_{10} or E_6. Even with the assumption that the low energy theory below M_X consists only of the MSSM fields, the resulting allowed region for the leptonic charges and the sparticle spectrum is quite different from the previous case.

We also derive some mass sum rules independent of the U_1' charges for this case, similar to the sum rules given in Refs. [17, 23].

2 The General Case

First of all, for completeness and to establish notation, let us recapitulate some standard results. We take an $N = 1$ supersymmetric gauge theory with gauge group $\Pi_\alpha G_\alpha$ and with superpotential

$$W(\Phi) = \tfrac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \tfrac{1}{2}\mu^{ij}\Phi_i\Phi_j. \tag{1}$$

We also include the standard soft supersymmetry-breaking terms

$$L_{\rm SB} = -(m^2)^j_i\phi^i\phi_j - \left(\tfrac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{2}M\lambda\lambda + \text{h.c.}\right) \tag{2}$$

where $\phi^i = (\phi_i)^*$.

For the moment let us assume that the gauge group has one abelian factor, which we shall take to be G_1. We shall denote the hypercharge matrix for G_1 by $\mathcal{Y}^i{}_j = \mathcal{Y}^j\delta^i{}_j$ and its gauge coupling by g_1.

At one loop we have

$$16\pi^2\beta_{g_\alpha}^{(1)} = g_\alpha^3 Q_\alpha = g_\alpha^3\left[T(R_\alpha) - 3C(G_\alpha)\right], \tag{3a}$$

$$16\pi^2\gamma^{(1)i}{}_j = P^i{}_j = \tfrac{1}{2}Y^{ikl}Y_{jkl} - 2\sum_\alpha g_\alpha^2[C(R_\alpha)]^i{}_j. \tag{3b}$$

Here β_{g_α} are the gauge β-functions and γ is the chiral supermultiplet anomalous dimension, R_α is the group representation for G_α acting on the chiral fields, $C(R_\alpha)$ the corresponding quadratic Casimir and $T(R_\alpha) = (r_\alpha)^{-1}\text{Tr}[C(R_\alpha)]$, r_α being the dimension of G_α. For the adjoint representation, $C(R_\alpha) = C(G_\alpha)I_\alpha$, where I_α is the $r_\alpha \times r_\alpha$ unit matrix. Obviously $T(R_1) = \text{Tr}[\mathcal{Y}^2]$, $[C(R_1)]^i{}_j = (\mathcal{Y}^2)^i{}_j$ and $C(G_1) = 0$. At two loops we have

$$(16\pi^2)^2\beta_{g_\alpha}^{(2)} = 2g_\alpha^5 C(G_\alpha)Q_\alpha - 2g_\alpha^3 r_\alpha^{-1}\text{Tr}\left[PC(R_\alpha)\right], \tag{4}$$

$$(16\pi^2)^2\gamma^{(2)i}{}_j = 2\sum_\alpha g_\alpha^4 C(R_\alpha)^i{}_j Q_\alpha - \left[Y_{jmn}Y^{mpi} + 2\sum_\alpha g_\alpha^2 C(R_\alpha)^p{}_j\delta^i{}_n\right]P^n{}_p. \tag{5}$$

The one-loop β-functions for the soft-breaking couplings are given by

$$16\pi^2\beta_h^{(1)ijk} = U^{ijk} + U^{kij} + U^{jki}, \tag{6a}$$

$$16\pi^2\beta_b^{(1)ij} = V^{ij} + V^{ji}, \tag{6b}$$

$$16\pi^2[\beta_{m^2}^{(1)}]^i{}_j = W^i{}_j, \tag{6c}$$

$$16\pi^2\beta_{M_\alpha}^{(1)} = 2g_\alpha^2 Q_\alpha M_\alpha, \tag{6d}$$

where

$$
\begin{aligned}
U^{ijk} &= h^{ijl}P^k{}_l + Y^{ijl}X^k{}_l, \\
V^{ij} &= b^{il}P^j{}_l + \tfrac{1}{2}Y^{ijl}Y_{lmn}b^{mn} + \mu^{il}X^j{}_l, \\
W^j{}_i &= \tfrac{1}{2}Y_{ipq}Y^{pqn}(m^2)^j{}_n + \tfrac{1}{2}Y^{jpq}Y_{pqn}(m^2)^n{}_i + 2Y_{ipq}Y^{jpr}(m^2)^q{}_r \\
&\quad + h_{ipq}h^{jpq} - 8\sum_\alpha g_\alpha^2 M_\alpha M_\alpha^* C(R_\alpha)^j{}_i,
\end{aligned} \tag{7}
$$

with

$$
X^i{}_j = h^{ikl}Y_{jkl} + 4\sum_\alpha g_\alpha^2 M_\alpha C(R_\alpha)^i{}_j. \tag{8}
$$

We have excluded from Eq. (6c) a D-tadpole contribution which arises if we calculate with the auxiliary field D eliminated. If we work in the D-eliminated form of the theory then we have instead of Eq. (6c):

$$
16\pi^2[\beta_{m^2}^{(1)}]^i{}_j \to W^i{}_j + 2g^2\mathcal{Y}^i{}_j\mathrm{Tr}[\mathcal{Y}m^2]. \tag{9}
$$

This extra contribution is only nonvanishing in a theory whose gauge group has an abelian factor. It can be equivalently viewed as a renormalisation of the Fayet-Iliopoulos parameter, as we shall now describe.

In $N = 1$ supersymmetric gauge theories whose gauge group has an abelian factor, there exists a possible invariant that is not otherwise allowed: the Fayet-Iliopoulos D-term,

$$
L = \xi \int V(x,\theta,\bar{\theta})\, d^4\theta = \xi D(x). \tag{10}
$$

The significance of the ξ term is of course well known. The part of the scalar potential dependent on the U_1 D-field is

$$
V_D = -\tfrac{1}{2}D^2 - D\left(\xi + g_1\phi_i\mathcal{Y}^i{}_j\phi^j\right), \tag{11}
$$

which upon elimination of the auxiliary field D becomes

$$
V_D = \tfrac{1}{2}(\xi + g_1\phi_i\mathcal{Y}^i{}_j\phi^j)^2, \tag{12}
$$

so that to obtain a supersymmetric ground state we require at least one field ϕ^i to have a charge with the opposite sign to ξ, and to develop a vacuum expectation value. Thus for supersymmetry to be unbroken on the scale set by ξ it is necessarily the case that the corresponding U_1 is spontaneously broken. In Ref. [23] we showed that in the presence of anomaly mediation soft supersymmetry-breaking terms it is quite natural for the U_1 symmetry to be broken at a large scale characterised by ξ while all scalars receive, from the U_1 D-term, (mass)^2 contributions characterised by the gravitino (or anomaly mediation) mass.

In previous papers [25]– [27] we have discussed the renormalisation of ξ in the presence of the soft terms. The result for β_ξ is as follows:

$$
\beta_\xi = \frac{\beta_g}{g}\xi + \hat{\beta}_\xi \tag{13}
$$

where $\hat{\beta}_\xi$ is determined by V-tadpole (or in components D-tadpole) graphs, and is independent of ξ.

We found that

$$16\pi^2\hat{\beta}_\xi^{(1)} = 2g_1 \mathrm{Tr}\left[\mathcal{Y}m^2\right], \tag{14}$$

$$16\pi^2\hat{\beta}_\xi^{(2)} = -4g_1 \mathrm{Tr}\left[\mathcal{Y}m^2\gamma^{(1)}\right]. \tag{15}$$

The three-loop contribution was computed in Ref. [26] for an abelian theory and for the MSSM in Ref. [27].

3 The AM Solution

Remarkably the following results are RG invariant [8]:

$$M_\alpha = m_0\beta_{g_\alpha}/g_\alpha, \tag{16a}$$

$$h^{ijk} = -m_0\beta_Y^{ijk}, \tag{16b}$$

$$(m^2)^i{}_j = \frac{1}{2}m_0^2\mu\frac{d}{d\mu}\gamma^i{}_j, \tag{16c}$$

$$b^{ij} = \kappa m_0\mu^{ij} - m_0\beta_\mu^{ij}. \tag{16d}$$

Here β_Y is the Yukawa β-function, given by

$$\beta_Y^{ijk} = \gamma^i{}_l Y^{ljk} + \gamma^j{}_l Y^{ilk} + \gamma^k{}_l Y^{ijl}, \tag{17}$$

with a similar expression for β_μ^{ij}. It must be emphasised that the RG invariance of Eq. (16c) holds in the D-uneliminated theory. That is to say, given Eq. (16a)-(16d) it follows that

$$\beta_{m^2} = \frac{1}{2}m_0^2\mu\frac{d}{d\mu}\left(\mu\frac{d}{d\mu}\gamma\right) \tag{18}$$

where in Eq. (18), β_{m^2} does not include D-tadpole contributions (that is, at one loop it is given by Eq. (6c)); the renormalisation of these is dealt with separately by β_ξ, as described in the last section.

Note the arbitrary parameter κ in Eq. (16d); its presence means that we can, in the MSSM, follow the usual procedure whereby the Higgs B-parameter is determined (along with the μ-term) by the electroweak minimisation. How *natural* is this procedure is an obvious question, to which we will return later.

The approach to the AM tachyonic slepton problem that we will follow is based on the fact that RG invariance is preserved if we replace $(m^2)^i{}_j$ in Eq. (16c) by

$$(\overline{m}^2)^i{}_j = \frac{1}{2}m_0^2\mu\frac{d}{d\mu}\gamma^i{}_j + k'(\mathcal{Y}')^i{}_j, \tag{19}$$

where k' is a constant and $\mathcal{Y}'$ is a matrix satisfying

$$(\mathcal{Y}')^i{}_l Y^{ljk} + (\mathcal{Y}')^j{}_l Y^{ilk} + (\mathcal{Y}')^k{}_l Y^{ijl} = 0 \tag{20}$$

Q	u^c	d^c	H_1	H_2	ν^c
$-\frac{1}{3}L$	$-e-\frac{2}{3}L$	$e+\frac{4}{3}L$	$-e-L$	$e+L$	$-2L-e$

Table 1: Anomaly free U_1 charges for arbitrary lepton doublet and singlet charges L and e respectively. U_1^{SM} corresponds to $L=-1/2$ and $e=1$.

and

$$\mathrm{Tr}\left[\mathcal{Y}'C(R_\alpha)\right]=0, \tag{21}$$

in other words $\mathcal{Y}'$ is a hypercharge matrix corresponding to a U_1 symmetry (which we shall denote U_1') with no mixed anomalies with the SM gauge group. This U_1' may in general be gauged, or a global symmetry.

The MSSM (including right-handed neutrinos) admits two independent generation-blind anomaly-free U_1 symmetries. The possible charge assignments are shown in Table 1.

Of course the $k'\mathcal{Y}'$ term in Eq. (19) corresponds in form to a FI D-term; we shall assume that in fact the associated U_1' gauge symmetry is broken at high energy and that the above contributions to the scalar masses are the only relic of this breaking that survive in the low energy effective field theory. That this is a perfectly natural scenario was demonstrated in Ref. [23].

Now let us consider a possible FI term ξD associated with the SM (or MSSM) U_1, U_1^{SM}. Here ξ is an independent parameter respecting all the symmetries of the MSSM; in the vast majority of analyses using, for example, CMSSM boundary conditions at gauge unification, it is assumed to be zero there. (For an exception, in which ξ is treated as an extra independent parameter at low energy, see Ref. [**?**]). Working in the D-eliminated formalism, the effect of radiative generation of an FI term as we run down to low scales is then automatically taken care of by the term added in Eq. (9) (and corresponding terms at higher loops). If, on the other hand we work with the D-uneliminated formalism then obviously if we assume ξ is zero at gauge unification then it is calculable at low energies using β_ξ from Eqs. (14,15). The resulting additional contributions to the masses from Eq. (12) will of course lead to precisely the same results for the masses as obtained directly from the running of the masses using the D-eliminated formalism.

How large the radiatively generated ξ is depends on the boundary conditions we assume for the scalar masses at gauge unification. Let us consider first the standard CMSSM (or MSUGRA) picture. In that case it is clear that with the assumption of a common scalar mass at gauge unification, $\beta_\xi^{(1)}$ vanishes there because U_1^{SM} is free of gravitational anomalies:

$$\mathrm{Tr}[\mathcal{Y}]=0. \tag{22}$$

Moreover, and less obviously, $\beta_\xi^{(1)}$ is in fact RG invariant; that is, using Eq. (6c) in Eq. (23) we find that

$$\mathrm{Tr}\left[\mathcal{Y}\beta_{m^2}^{(1)}\right]=0 \tag{23}$$

where we denote the SM hypercharge by $\mathcal{Y}$. This follows because $\mathcal{Y}$ naturally satisfies Eq. (20),

(with $\mathcal{Y}'$ replaced by $\mathcal{Y}$):

$$\mathcal{Y}^i{}_l Y^{ljk} + \mathcal{Y}^j{}_l Y^{ilk} + \mathcal{Y}^k{}_l Y^{ijl} = 0 \tag{24}$$

(similarly for h^{ijk}) and anomaly cancellation,

$$\mathrm{Tr}\left[\mathcal{Y}C(R_\alpha)\right] = 0. \tag{25}$$

So for CMSSM boundary conditions, or indeed *any* boundary conditions such that $\mathrm{Tr}\left[\mathcal{Y}m^2\right] = 0$ at gauge unification, then, in the one-loop approximation, ξ is zero at low energy if it is zero at gauge unification. (If we go beyond one loop then a non-zero but quite small ξ will be generated.)

We turn now to the AM scenario. Substituting Eq. (19) in Eq. (14) and Eq. (15) we find that up to two loops we can write

$$16\pi^2\hat{\beta}_\xi = g_1|m_0|^2\left(\mu\frac{d}{d\mu}\mathrm{Tr}[\mathcal{Y}(\gamma-\gamma^2)] + 2k'\mathrm{Tr}[\mathcal{Y}\mathcal{Y}'(1-2\gamma)]\right), \tag{26}$$

and since gauge invariance and anomaly cancellation combined with Eqs. (3b) and (5) yield [25]

$$\mathrm{Tr}[\mathcal{Y}\gamma^{(1)}] = \mathrm{Tr}[\mathcal{Y}(\gamma^{(2)} - (\gamma^{(1)})^2)] = 0, \tag{27}$$

this reduces to

$$16\pi^2\hat{\beta}_\xi = 2k'g_1|m_0|^2\mathrm{Tr}[\mathcal{Y}\mathcal{Y}'(1-2\gamma)]. \tag{28}$$

Thus in the *absence* of the $\mathcal{Y}'$ term (i.e. with the unmodified mass solution of Eq. (16c)) an appreciable U_1^{SM} FI term will not be generated by the running, and Eq. (16c) will therefore be RG invariant. This was the conclusion of Ref. [27].

Using Eq. (19) however, we obtain Eq. (28), which is non-vanishing even at leading order unless we choose the charges $\mathcal{Y}'$ so that

$$\mathrm{Tr}[\mathcal{Y}\mathcal{Y}'] = 0. \tag{29}$$

This was in fact the choice made in Ref. [17], the motive there being to suppress kinetic mixing between the U_1^{SM} and the U_1' gauge bosons (in that paper we considered a U_1' broken at rather lower energies). With such a U_1', the $\mathcal{Y}'$ charges L and e satisfy

$$3L + 7e = 0 \tag{30}$$

so they are opposite in sign. Consequently Eq. (19) alone would not suffice to escape the tachyonic slepton problem (if it held at low energy). In Ref. [17] it was shown, however, that replacing Eq. (19) by

$$(\overline{m}^2)^i{}_j = \frac{1}{2}m_0^2\mu\frac{d}{d\mu}\gamma^i{}_j + k(\mathcal{Y}^{SM})^i{}_j + k'(\mathcal{Y}')^i{}_j, \tag{31}$$

(with $\mathcal{Y}'$ charges satisfying Eq. (30)) could do so. Now since we have shown above that an effective U_1^{SM} FI-term is in any event generated by RG running, it is not a priori obvious that having simply Eq. (19) *at gauge unification* even with a U_1' with opposite L, e charges won't work; however we may expect that the U_1' choice of Ref. [17] clearly will not do, precisely because of Eq. (29); the generated ξ for U_1^{SM} will be too small. We shall see that this is indeed the case.

One might hope that it would be possible to choose, for example, $U_1' \equiv U_1^{B-L}$; we shall see, however, that, with Eq. (19), although the region of (e, L) parameter space corresponding to an acceptable supersymmetric spectrum *does* indeed include the possibility of $L < 0$, it permits neither Eq. (29) nor $L + e = 0$, which would have corresponded to U_1^{B-L}.

Let us now follow Ref. [23] by considering a theory with FI-type contributions associated with U_1', and compare the consequences of imposing Eq. (19) (and vanishing FI term for U_1^{SM}) at (i) gauge unification (ii) a common SUSY scale, M_{SUSY}. It should be clear from the above discussion that using the same values of (e, L) in the two cases will not give rise to the same spectrum, because imposing it at gauge unification (say) will give rise to a non-vanishing U_1^{SM} FI term at M_{SUSY}, and corresponding contributions to the sparticle masses.

It is easy to see, however, that precisely the same spectrum consequent on a particular choice of (e, L) at at M_X can be obtained by by using a *different* (e, L) pair at M_{SUSY} (with in each case no U_1^{SM} FI term). This is simply because we can write

$$\begin{aligned}
\overline{m}_L^2 &= m_L^2 - \tfrac{1}{2}k + k'L = m_L^2 + k''L'' \\
\overline{m}_{e^c}^2 &= m_{e^c}^2 + k + k'e = m_{e^c}^2 + k''e'' \\
\overline{m}_Q^2 &= m_Q^2 + \tfrac{1}{6}k + k'Q = m_Q^2 + k''Q'' \quad \text{etc.,}
\end{aligned} \tag{32}$$

where $k''Q'' = -k''\frac{1}{3}L''$, etc.

Thus we can absorb the U_1^{SM} FI term generated by the running into a redefinition of the charges (e, L).

Note that the above remarks strictly apply only if we evaluate the spectrum at a common mass scale, M_{SUSY}. Since in Ref. [23] we systematically evaluated each sparticle pole mass at a renormalisation scale equal to the pole mass itself, small discrepancies were introduced. From now on we will always calculate spectra by running down from M_X, inputting (e, L) (and zero for the U_1^{SM} FI term) there.

4 The MSSM and the sparticle spectrum

The MSSM is defined by the superpotential:

$$W = H_2 Q Y_t t^c + H_1 Q Y_b b^c + H_1 L Y_\tau \tau^c + \mu H_1 H_2 \tag{33}$$

with soft breaking terms:

$$\begin{aligned}
L_{\text{SOFT}} &= \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + \text{h.c.} \right] \\
&+ \left[H_2 Q h_t t^c + H_1 Q h_b b^c + H_1 L h_\tau \tau^c + \text{h.c.} \right]
\end{aligned} \tag{34}$$

where in general $Y_{t,b,\tau}$ and $h_{t,b,\tau}$ are 3×3 matrices. We work throughout in the approximation that the Yukawa matrices are diagonal, and neglect the Yukawa couplings of the first two generations.

The anomalous dimensions of the Higgses and 3rd generation matter fields are given (at one loop) by

$$
\begin{aligned}
16\pi^2\gamma_{H_1} &= 3\lambda_b^2 + \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{H_2} &= 3\lambda_t^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{L} &= \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{Q} &= \lambda_b^2 + \lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{3}{2}g_2^2 - \tfrac{1}{30}g_1^2, \\
16\pi^2\gamma_{t^c} &= 2\lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{8}{15}g_1^2, \\
16\pi^2\gamma_{b^c} &= 2\lambda_b^2 - \tfrac{8}{3}g_3^2 - \tfrac{2}{15}g_1^2, \\
16\pi^2\gamma_{\tau^c} &= 2\lambda_\tau^2 - \tfrac{6}{5}g_1^2,
\end{aligned}
\tag{35}
$$

where $\lambda_{t,b,\tau}$ are the third generation Yukawa couplings. For the first two generations we use the same expressions but without the Yukawa contributions. The two and three loop results for the anomalous dimensions and the gauge β-functions may be found in Ref. [29].

The soft scalar masses are given by

$$
\begin{aligned}
\overline{m}_Q^2 &= m_Q^2 - \tfrac{1}{3}Lk', \quad \overline{m}_{t^c}^2 = m_{t^c}^2 - (\tfrac{2}{3}L + e)k', \\
\overline{m}_{b^c}^2 &= m_{b^c}^2 + (\tfrac{4}{3}L + e)k', \quad \overline{m}_L^2 = m_L^2 + Lk', \\
\overline{m}_{\tau^c}^2 &= m_{\tau^c}^2 + ek', \quad \overline{m}_{H_{1,2}}^2 = m_{H_{1,2}}^2 \mp (e+L)k',
\end{aligned}
\tag{36}
$$

(with similar expressions for the first two generations) where m_Q^2 etc are the pure anomaly-mediation contributions, for example:

$$
m_Q^2 = \tfrac{1}{2}m_0^2\mu\frac{d}{d\mu}\gamma_Q = \tfrac{1}{2}m_0^2\beta_i\frac{\partial}{\partial\lambda_i}\gamma_Q \tag{37}
$$

(here λ_i includes all gauge and Yukawa couplings) and k' is the effective FI parameter.

The 3rd generation A-parameters are given by

$$
\begin{aligned}
A_t &= -m_0(\gamma_Q + \gamma_{t^c} + \gamma_{H_2}), \\
A_b &= -m_0(\gamma_Q + \gamma_{b^c} + \gamma_{H_1}), \\
A_\tau &= -m_0(\gamma_L + \gamma_{\tau^c} + \gamma_{H_1})
\end{aligned}
\tag{38}
$$

and we set the corresponding first and second generation quantities to zero. The gaugino masses are given by

$$
M_\alpha = m_0\left(\frac{\beta_{g_\alpha}}{g_\alpha}\right), \quad \text{for} \quad \alpha = 1,2,3. \tag{39}
$$

The manner in which the scale of the effective FI parameter contributions $k'L$ etc. to the sparticle masses can naturally be of the same order as the anomaly mediation contributions when a U_1' is broken at high energies is explained in Ref. [23] and Ref. [24].

Clearly these FI contributions depend on two parameters, Lk' and ek'. For notational simplicity we will set $k' = 1(\text{TeV})^2$ from now on.

We begin by choosing input values for m_0, $\tan\beta$, L, e and signμ at M_X and then we calculate the appropriate dimensionless coupling input values at the scale M_Z by an iterative procedure

involving the sparticle spectrum, and the loop corrections to $\alpha_{1\cdots3}$, m_t, m_b and m_τ, as described in Ref. [30]. We define gauge unification by the meeting point of α_1 and α_2. For the top quark pole mass we use $m_t = 170.9$GeV.

We then determine a given sparticle pole mass by running the dimensionless couplings up to a certain scale chosen (by iteration) to be equal to the pole mass itself, and then using Eqs. (37), (38), (39) and including full one-loop corrections from Ref. [30], and two-loop corrections to the top quark mass [31].

As in Ref. [32], we have compared the effect of using one, two and three-loop anomalous dimensions and β-functions in the calculations. Note that when doing the three-loop calculation, we use in Eq. (37), for example, the three loop approximation for both β_i and γ_Q, thus including some higher order effects.

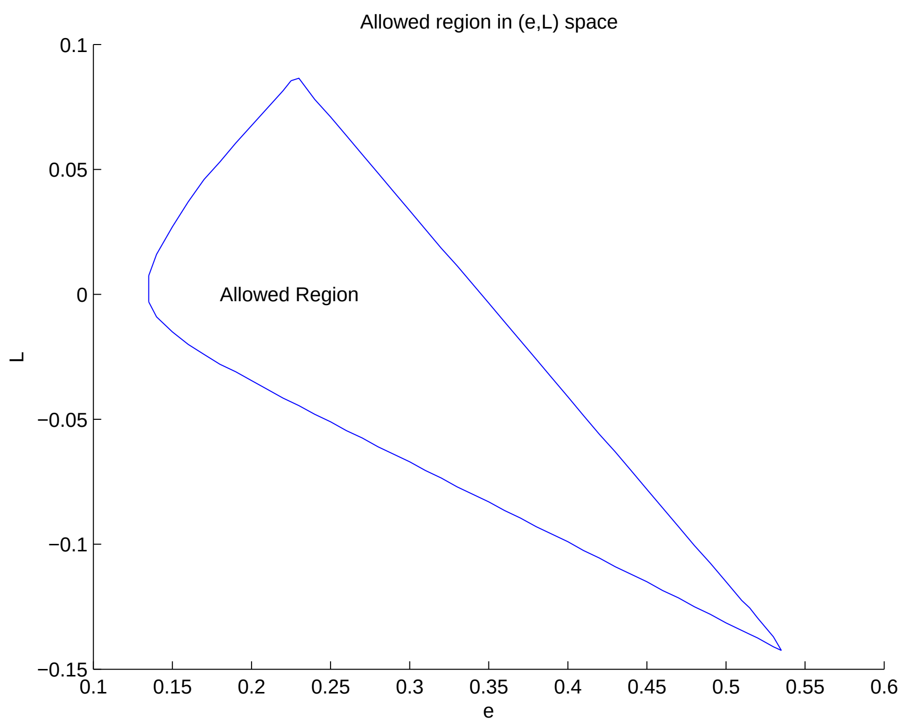

Figure 1: The region of (e, L) space corresponding to an acceptable electroweak vacuum, for $m_0 = 40$TeV and $\tan\beta = 10$.

The allowed region in (e, L) space for $\mu > 0$ and $m_0 = 40$TeV corresponding to an acceptable vacuum is shown in Fig. 1. To define the allowed region, we have imposed $m_{\tilde{\tau}} > 82$GeV, $m_{\tilde{\nu}_\tau} > 49$GeV and $m_A > 90$GeV. The region is to a good approximation triangular, with one side of the triangle corresponding to m_A becoming too light (and quickly imaginary just beyond the boundary, with breakdown of the electroweak vacuum) and the other two sides to one of the sleptons (usually a stau) becoming too light.

Note that as we remarked earlier, the allowed region includes parts with $L < 0$. To understand this, consider, for example, the point $(e, L) = (0.35, -0.05)$. For this point we find

mass (GeV)	1loop	2loops	3loops
$\tilde{g}$	925	900	897
$\tilde{t}_1$	766	757	746
$\tilde{t}_2$	502	500	487
$\tilde{u}_L$	834	819	808
$\tilde{u}_R$	774	766	753
$\tilde{b}_1$	724	712	702
$\tilde{b}_2$	956	946	936
$\tilde{d}_L$	838	822	812
$\tilde{d}_R$	965	955	946
$\tilde{\tau}_1$	267	266	266
$\tilde{\tau}_2$	212	199	199
$\tilde{e}_L$	262	261	262
$\tilde{e}_R$	225	212	212
$\tilde{\nu}_e$	250	249	249
$\tilde{\nu}_\tau$	248	247	247
χ_1	106	131	131
χ_2	354	362	362
χ_3	569	593	585
χ_4	580	604	596
$\chi_1^\pm$	107	131	131
$\chi_2^\pm$	577	601	594
h	114	114	114
H	333	373	361
A	333	373	361
$H^\pm$	342	381	370
$\chi_1^\pm - \chi_1$ (MeV)	226	235	237

Table 2: Mass spectrum for $m_t = 170.9\text{GeV}$, $m_0 = 40\text{TeV}$, $\tan\beta = 10$, $L = 0$, $e = 1/4$

that

$$\text{Tr}[\mathcal{Y}\mathcal{Y}'] = 4.6. \tag{40}$$

This is positive so from Eq. (28) we see that β_ξ for the U_1^{SM} FI term is positive at M_X. Since we are running *down* from M_X it follows that a *negative* ξ^{SM} is generated, and hence a *positive* contribution to $\overline{m}_L^2$, since the U_1^{SM} charge of the lepton doublet is negative. Evidently the same reasoning means that we cannot have $e < 0$ at M_X, as we indeed see to be the case.

Although $L < 0$ is allowed, it is easily seen that we cannot, as we mentioned earlier, have either $L + e = 0$ (corresponding to U_1^{B-L}) or $3L + 7e = 0$ (corresponding to $\text{Tr}[\mathcal{Y}\mathcal{Y}'] = 0$).

As an example of an acceptable spectrum, we give in Table 2 the results for $m_0 = 40\text{TeV}, \tan\beta = 10, L = 0, e = 1/4, \text{sign}\mu = +$ as derived using the one, two and three loop approximations for the anomalous dimensions and β-functions.

This point in (e, L) space is near the centre of the allowed region (see Fig. 1). As explained

in the previous section, the same spectrum would be obtained to a good approximation by inputting parameters and calculating pole masses at M_Z with a different pair of (e, L) values, in this case $(e, L) \approx (0.06, 0.09)$. This point is near the centre of the allowed region in Fig. 1 of Ref. [23]. In Table 2, however, we give the masses with each calculated at a scale equal to its pole mass. Therefore as explained before, this means the whole spectrum corresponds to choosing the FI U_1^{SM} term to be zero at M_Z, but to a set of (e, L) close to but each differing slightly from $(0.06, 0.09)$.

For the choice of parameters leading to Table 2, we find that $\mu \sim 576\text{GeV}$ and $B \sim (140\text{GeV})^2$, leading to $\kappa \sim 0.008$. We see that, aside from the little hierarchy problem associated with the fact that $\mu >> M_Z$, we have the problem of accounting for the small value of κ, and a degree of fine tuning between the two terms in Eq. (16d). As in Ref. [23] we find that to obtain a sufficiently high light CP-even Higgs mass, m_h and an electroweak vacuum we need to have $25 \gtrsim \tan\beta \gtrsim 8$.

For discussion of AM characteristic phenomenology the reader is referred to Refs. [1]- [24], and in particular Ref. [4].

5 U_1 and GUTs

In the previous sections we have been assuming that our theory has gauge group $\mathcal{G}_{SM} \otimes U_1'$, broken to $\mathcal{G}_{SM}$ at high energies. Let us now ask what modifications ensue if we ask for compatibility with a simple GUT embedding; for definiteness let us take SU_5, and imagine that our matter fields form a set of n_f $(\overline{5} + 10)$ multiplets as usual, and promote our Higgs multiplets to n_h sets of $(5 + \overline{5})$. Then for compatibility with an $SU_5 \otimes U_1'$ embedding we at once have the relations

$$\begin{aligned} Q &= u^c = e \\ d^c &= L \end{aligned} \tag{41}$$

and for U_1' invariance of the Yukawa terms

$$\begin{aligned} h_1 &= -L - e \\ h_2 &= -2e \\ \nu^c &= 2e - L. \end{aligned} \tag{42}$$

Then the $SU_3^2 \otimes U_1'$, $SU_2^2 \otimes U_1'$ and $(U_1^{SM})^2 \otimes U_1'$ anomalies are all proportional to the quantity

$$A_1 = (n_f - n_h)(L + 3e) \tag{43}$$

while the $(U_1')^2 \otimes U_1^{SM}$ anomaly vanishes. The $(U_1')^3$ anomaly is proportional to

$$A_3 = (L + 3e)\left[5(n_f - n_h)(L^2 + 3e^2) - n_f(L + 3e)^2\right] \tag{44}$$

while the U_1' − gravitational anomaly is proportional to

$$A_G = (L + 3e)(4n_f - 5n_h). \tag{45}$$

Thus if $L + 3e = 0$ then the $\mathcal{G}_{SM} \otimes U_1'$ theory is anomaly-free for arbitrary n_f, n_h. This special case corresponds in fact to compatibility with the embedding $SO_{10} \supset SU_5 \otimes U_1'$ with each set

10	$\overline{5}$	ν^c	H	$\overline{H}$	N
e	L	$2e-L$	$-2e$	$-e-L$	$L+3e$

Table 3: Anomaly free U_1 symmetry for arbitrary lepton doublet and singlet charges

of matter fields forming a 16 and each set of Higgs fields a 10 under SO_{10}. (Although SO_{10} has complex representations they are all anomaly-free). Note the opposite sign charges for L and e; we argued in Section 3 that this does not preclude starting from M_X with an FI term for such a U_1', but we shall see that the line $L+3e=0$ does not cross the allowed (e,L) region for our class of models. The other way to produce an anomaly-free theory is to first set $n_h = n_f$. Then $A_1 = 0$ while for A_3 and A_G we have

$$\begin{aligned} A_3 &= -n_f(L+3e)^3 \\ A_G &= -n_f(L+3e) \end{aligned} \tag{46}$$

so that we can obtain an anomaly-free theory by adding a further set of n_f $\mathcal{G}_{SM}$-singlet fields N, with charges $L+3e$. The resulting charge assignments are shown in Table 3.

This structure is compatible with $SU_5 \otimes U_1'$, and can be embedded in E_6, when Table 3 forms a 27. (Recall that E_6 also has only anomaly-free representations). If $L=e$ we could have $E_6 \supset SO_{10} \otimes U_1'$, (with Table 3 forming a $16 \oplus 10 \oplus 1$ of SO_{10}), or, as explained above, for $L=-3e$ we could have $SO_{10} \supset SU_5 \otimes U_1'$. Another possibility is to have $L=2e$, in order that ν^c have zero U_1' charge [33]; evidently this would have model-building advantages if one wants to have a large mass for ν^c while breaking U_1' at lower energy. Of course one sees easily that the cases $L=-3e$ and $L=2e$ are equivalent from a group theoretic point of view under the exchanges $N \leftrightarrow \nu^c$ and $\overline{5} \leftrightarrow \overline{H}$; obviously in the latter case we could have an anomaly-free theory with n_f sets of $(10, \overline{H}, N)$ and n_h sets of $(H, \overline{5})$.

Let us now suppose that, whatever the nature of the underlying theory, below gauge unification we have the usual MSSM effective field theory, with three generations and a single pair of Higgs doublets (of course an explicit construction may lead to a more exotic low energy theory, but here we will confine ourselves to this possibility). We also assume FI contributions to the sparticle masses corresponding to our new U_1', thus instead of Eq. (36) we have:

$$\begin{aligned} \overline{m}_Q^2 &= m_Q^2 + ek', \quad \overline{m}_{t^c}^2 = m_{t^c}^2 + ek', \quad \overline{m}_{\tau^c}^2 = m_{\tau^c}^2 + ek', \\ \overline{m}_{b^c}^2 &= m_{b^c}^2 + Lk', \quad \overline{m}_L^2 = m_L^2 + Lk', \\ \overline{m}_{H_1}^2 &= m_{H_1}^2 - (e+L)k', \qquad \overline{m}_{H_2}^2 = m_{H_2}^2 - 2ek', \end{aligned} \tag{47}$$

where m_Q^2 etc are again the pure anomaly-mediation contributions, and once again we set $k'=1$.

We can then compare the predicted sparticle spectrum with that obtained in the last section. We may expect there to be differences, since evidently if we have both $(e,L) > 0$ it is now the case that both squarks and sleptons will have positive (mass)^2 contributions. We calculate the

spectrum as described in the previous section, running down from M_X; of course RG invariance of the AM masses no longer holds because the effective field theory is no longer anomaly-free with respect to the U_1'.

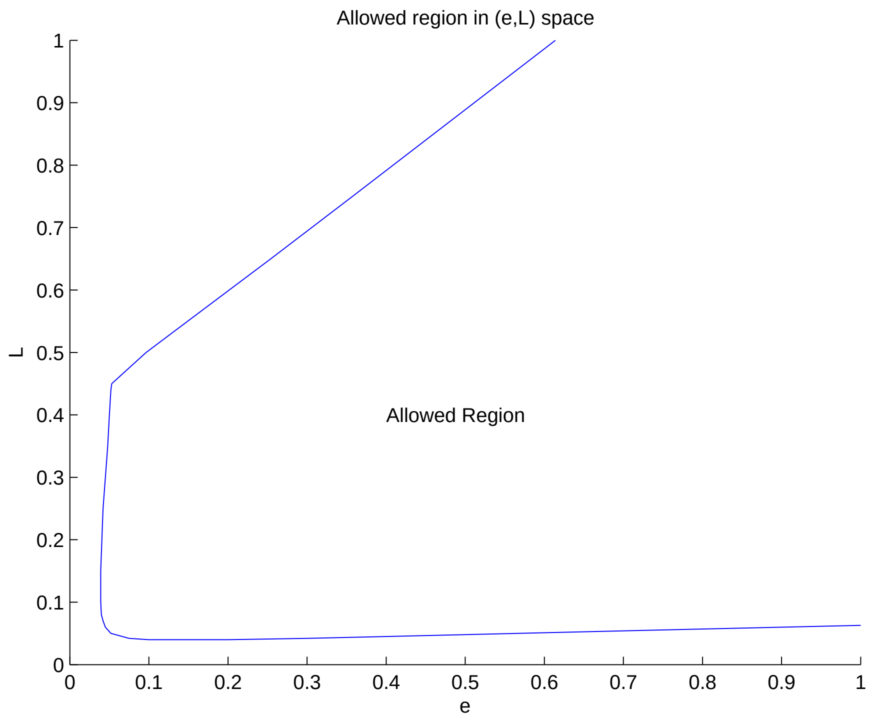

Figure 2: The region of (e, L) space corresponding to an acceptable electroweak vacuum, for $m_0 = 40$TeV and $\tan\beta = 15$.

The allowed (e, L) region with our new charge assignments is shown in Fig. 2. Comparing with Fig. 1, we see that the most dramatic difference is that increasing (e, L) does not lead to loss of the electroweak vacuum as long as $L \lesssim e + 0.4$. Of course increasing (e, L) scales up the squark and slepton masses, $|m_{H_{1,2}^2}|$ and hence the (Higgs) μ-parameter, thus increasing the fine-tuning known as the little hierarchy problem. Other scenarios explored recently have also had this feature, for example split supersymmetry [34], and the G_2 based model of Ref. [35]. For a recent discussion of the little hierarchy problem see (for example) Ref. [36].

Another distinctive feature of the new charge assignment is that acceptable spectra are obtained with larger values of $\tan\beta$ than in section 4; here we find an upper limit of $\tan\beta = 43$.

In Table 4 we give results for the sparticle spectrum for a representative point in the allowed region. Of course $L = 1/3$ and $e = 1/2$ represent significant contributions to the squark squared masses, which are in any case already positive in AM, so it is not surprising that these masses are quite large for this point. Correspondingly the value of μ determined from electroweak minimisation is quite high at around 1TeV.

Both $L = e$ (corresponding to a potential $SO_{10} \otimes U_1'$ embedding) and $L = 2e$ (corresponding

mass (GeV)	1loop	2loops	3loops
$\tilde{g}$	966	940	938
$\tilde{t}_1$	1063	1047	1040
$\tilde{t}_2$	936	923	917
$\tilde{u}_L$	1103	1081	1073
$\tilde{u}_R$	1102	1085	1077
$\tilde{b}_1$	1038	974	1013
$\tilde{b}_2$	993	1021	966
$\tilde{d}_L$	1105	1084	1076
$\tilde{d}_R$	1032	1014	1005
$\tilde{\tau}_1$	550	544	544
$\tilde{\tau}_2$	698	697	697
$\tilde{e}_L$	556	551	551
$\tilde{e}_R$	698	697	697
$\tilde{\nu}_e$	550	545	545
$\tilde{\nu}_\tau$	548	542	543
χ_1	111	135	135
χ_2	362	369	369
χ_3	1204	1211	1207
χ_4	1206	1213	1209
$\chi_1^\pm$	111	135	136
$\chi_2^\pm$	1207	1214	1210
h	115	115	115
H	737	743	737
A	737	743	737
$H^\pm$	742	748	742
$\chi_1^\pm - \chi_1$ (MeV)	185	192	192

Table 4: Mass spectrum for $m_t = 170.9\text{GeV}$, $m_0 = 40\text{TeV}$, $\tan\beta = 15$, $L = 1/3$, $e = 1/2$

mass (GeV)	1loop	2loops	3loops
$\tilde{g}$	934	910	907
$\tilde{t}_1$	858	847	838
$\tilde{t}_2$	688	680	672
$\tilde{u}_L$	908	891	881
$\tilde{u}_R$	911	899	889
$\tilde{b}_1$	803	789	780
$\tilde{b}_2$	894	882	872
$\tilde{d}_L$	911	894	885
$\tilde{d}_R$	916	904	894
$\tilde{\tau}_1$	236	231	231
$\tilde{\tau}_2$	311	308	308
$\tilde{e}_L$	282	275	275
$\tilde{e}_R$	282	281	281
$\tilde{\nu}_e$	270	263	263
$\tilde{\nu}_\tau$	266	259	259
χ_1	109	133	134
χ_2	358	365	365
χ_3	820	833	828
χ_4	826	839	834
$\chi_1^\pm$	109	134	134
$\chi_2^\pm$	826	839	834
h	115	115	115
H	623	635	629
A	624	636	629
$H^\pm$	629	641	634
$\chi_1^\pm - \chi_1$ (MeV)	192	199	200

Table 5: Mass spectrum for $m_t = 170.9\text{GeV}$, $m_0 = 40\text{TeV}$, $\tan\beta = 15$, $L = e = 0.1$

to zero U_1' charge for ν^c) are allowed; in the latter case we would need to have $e \lesssim 0.4$. In Table 5 we give results for the sparticle spectrum for $L = e = 0.1$, while in Table 6 we give results for the sparticle spectrum for $L = 2e = 0.1$.

6 Mass sum rules

By taking appropriate linear combinations of squark and slepton (masses)^2 so that the (e, L) contributions cancel it is straightforward to derive a pair of interesting sum rules similar to those

mass (GeV)	1loop	2loops	3loops
$\tilde{g}$	930	906	903
$\tilde{t}_1$	828	818	809
$\tilde{t}_2$	650	642	633
$\tilde{u}_L$	880	864	854
$\tilde{u}_R$	884	873	863
$\tilde{b}_1$	771	759	749
$\tilde{b}_2$	893	882	872
$\tilde{d}_L$	883	868	857
$\tilde{d}_R$	916	905	895
$\tilde{\tau}_1$	290	285	285
$\tilde{\tau}_2$	131	126	127
$\tilde{e}_L$	284	278	278
$\tilde{e}_R$	165	162	162
$\tilde{\nu}_e$	272	266	266
$\tilde{\nu}_\tau$	268	262	262
χ_1	109	133	133
χ_2	358	365	365
χ_3	759	774	768
χ_4	766	780	775
$\chi_1^\pm$	109	133	134
$\chi_2^\pm$	765	780	775
h	115	115	115
H	585	599	591
A	585	599	592
$H^\pm$	591	604	597
$\chi_1^\pm - \chi_1$ (MeV)	195	203	203

Table 6: Mass spectrum for $m_t = 170.9$GeV, $m_0 = 40$TeV, $\tan\beta = 15$, $L = 2e = 0.1$

we derived [17, 23], with the original charge assignments of Section 4:

$$\begin{aligned}
m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 - m_{\tilde{u}_R}^2 - m_{\tilde{e}_R}^2 &\approx 0.8\,(m_{\tilde{g}})^2\,,\\
m_A^2 + \sec 2\beta\left(m_{\tilde{e}_R}^2 - m_{\tilde{e}_L}^2\right) - 2M_W^2 + \tfrac{5}{2}M_Z^2 &\approx 0.5\,(m_{\tilde{g}})^2\,,\\
m_{\tilde{b}_1}^2 + m_{\tilde{b}_2}^2 - m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2 &\approx 1.5\,(m_{\tilde{g}})^2\,,\\
m_{\tilde{b}_1}^2 + m_{\tilde{b}_2}^2 - m_{e_L}^2 - m_{e_R}^2 &\approx 1.5\,(m_{\tilde{g}})^2\,,\\
m_{d_L}^2 + m_{d_R}^2 - m_{e_L}^2 - m_{e_R}^2 &\approx 1.8\,(m_{\tilde{g}})^2\,. \qquad (48)
\end{aligned}$$

Although these sum rules are derived using the tree results for the various masses they hold reasonably well for the physical masses. The numerical coefficients on the RHS of Eq. (48) are in fact slowly varying functions of $\tan\beta$; the above results correspond to $\tan\beta = 15$.

7 Conclusions

The AM scenario is an attractive alternative to (and distinguishable from) the CMSSM. With AM it is possible to imagine a theory where the only explicit scale in the effective field theory is the gravitino mass. An explicit realisation of this idea was given in Ref. [24], where the scale corresponding to the spontaneous breaking of an additional U_1' symmetry (needed to solve the tachyonic slepton problem) was generated by dimensional transmutation. (This theory had the additional feature of a weakly coupled chiral matter multiplet whose fermionic component is a dark matter candidate). There is no obstacle in principle to extending this idea to a Grand Unified Theory, with the unification scale similarly generated by dimensional transmutation; this idea led us to consider the alternative charge assignments of section 5. One possibility would be a variation of the inverted hierarchy model of Witten [37], defined by the superpotential

$$W = \lambda_1 \mathrm{Tr}(A^2 Y) + \lambda_2 X(\mathrm{Tr}A^2 - m^2) \qquad (49)$$

where A, Y are SU_5 adjoints and X is a singlet. In its original form, supersymmetry is broken spontaneously in the O'Raifertaigh manner; moreover SU_5 is broken to $SU_3 \otimes SU_2 \otimes U_1$, with the scale at which this occurs being unrelated to m^2, and generated by dimensional transmutation. Our variation would be to have $m^2 = 0$ in Eq. (49), with the SU_5 breaking generated in similar fashion[2] but the supersymmetry breaking provided instead by anomaly mediation. We will explore this model in more detail elsewhere.

We have shown that while a U_1' gauge symmetry broken at high energies can lead in a natural way to the FI-solution to the AM tachyonic slepton problem, care must be taken with regard to the FI term associated with U_1^{SM}. We have also shown how an extension of the minimal model permits a gauged U_1' compatible with grand unification, with, in this case, sparticle spectra characterised by both heavy squarks and heavy sleptons.

[2] A discussion of the $m^2 \to 0$ limit of Witten's model appears in Ref. [38].

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